Session 5: Discussion Points

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In this discussion session we will review some background on selected topics and discuss the exercises and open problems below. The topics to be discussed include:

1. A more detailed introduction to CSPs, with examples;

2. a brief introduction to DataLog;

3. linear programming relaxations;

4. semi-definite programming and the Laserre Hierarchy;

5. new challenges for the duplicator;

6. CPT, higher-order objects, and lower-bounds; and

7. games for rank logic.

A Few Open Questions to be Discussed

1. Both FPC and FPR have symmetric circuit characterisations. It's natural to ask whether CPT admits a similar characterisation. The chief obstacle seems to be that the notion of symmetry considered for CPT seems less restrictive in some sense than the one studied in the context of symmetric circuits (and the various other models we've seen in this course). This fact itself is interesting, and so understanding (and perhaps overcoming) this barrier would likely be both beneficial for its own sake as well as aid in resolving other open questions.

2. Can we compute the rank of a matrix in CPT? This and numerous other questions regarding the limitations of CPT to solve problems from linear algebra (and, more generally, the theory of modules) would help enormously towards understanding CPT and settling the question of whether CPT contains FPR (and similarly defined logics).

3. We might ask: Can we study symmetry as a resource. One approach could be to weaken the symmetry condition on the circuit by considering the action of a subgroup of the symmetric group on the input gates and requiring only that those permutations from the subgroup extend to automorphisms of the circuit. Of course, taking this subgroup to be the trivial group gives a general circuit and taking it to be the full symmetric group gives the usual definition of a symmetric circuit. In this way we can interpolate between these two extremes and study symmetry more directly.

Exercises

1. Prove that FPC is contained in CPT with counting. What is the rank of the sets activated?

2. There is a well-known polynomial-time algorithm that can approximate the size of the minimum vertex cover of a graph up to a factor of 2. Show that this can be done in FPC.