Session 3: Discussion Points

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In this discussion session we will review some background on selected topics and discuss the exercises below. The topics to be discussed include:

1. a sketch of a proof of the result mentioned on slide 15 establishing that FPC and polynomial-time bounded relational machines have the same expressive power;
2. the details of the three reductions mentioned on slide 14;
3. some more discussion on supports and how we get exponential bounds for symmetric circuits; and
4. details on how bijection games can be played directly using the circuit, without need for reference to logic.

Exercises

1. Show that a tree has tree-width 1 and that a cycle has tree-width 2.
2. What is the tree-width of a complete graph?
3. Show that the $k \times h$ grid (i.e. the grid with k rows and h columns) has tree-width $\min(k, h)$.
4. Describe the automorphism groups of the graphs X_G and $\tilde{X}_G$ given on slide 3.
5. Prove the claim in the first sentence of slide 25 from the lecture.
6. Try and construct a polynomial-size family of symmetric circuits over the standard basis that decide the parity of the number of edges in a graph. We might try and start with the usual circuit for parity and then copy gates so as to force the circuit to be symmetric. What goes wrong? Construct a family of symmetric circuits over the standard basis with threshold gates that decides the same query.
7. It is sometimes helpful when working with (general) circuits with a polynomial bound on depth to assume all gates have fan-in two.
   (a) Why can this assumption be made for these circuits without a loss of generality?
   (b) Can we make a similar assumption for symmetric circuits without a loss of generality? [Hint: consider symmetric circuits where every internal gate has in-degree 2. Try and prove a lower bound for this model.]