Session 1: Discussion Points

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In this discussion session we will review some background on selected topics and discuss the exercises given here. The topics to be discussed include:

1. Ehrenfeucht–Fraïssé Games and FO definability,
2. complexity questions and FO,
3. background on conventional complexity classes, and
4. background on circuit complexity.

Exercises

1. Show that that completeness of a graph and the existence of an isolated vertex in a graph are FO definable properties.
2. Show that the existence of a Hamiltonian cycle and the property that the diameter of a graph is at most half the number of vertices are definable in SO. Which of these can we certainly say is definable in the existential fragment of SO? What about the universal fragment?
3. Show that the graph matching algorithm based on augmenting paths results in a maximum matching (see slide 12 of the lectures).
4. We saw that FO sentences may be translated to algorithms that run in time polynomial (and space logarithmic) in the size of the input structure. Show that if a class of graphs is SO definable then the corresponding decision problem is in PSPACE. Could we prove the result for a complexity class believed to be strictly contained in PSPACE?
5. We require that the logic on slide 27 have the property that the map from sentences to Turing machines in computable. Can we answer the conjecture if this requirement is dropped?
6. On slide 34 of the lecture we sketch a description of how to translate an FO formula to a circuit. Take any of the FO formulas defined in the lecture notes or above and write out the corresponding circuit. With this translation in mind, what does this tell you about the relationship between AC_0 and FO?